

VEDIC MATH – MULTIPLICATION

by

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“The Vedic Math Guy”

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Veda, by definition, is ‘knowledge’. Hence Vedic Math has a much ancient origin though attributed to the techniques rediscovered between 1911-1918 (see January 2003 column in Lotus). Mathematicians from across the spectrum from Hindu, Buddha and Jaina sub-cultures have contributed immensely to this body of knowledge. To learn about Vedic Math in these columns, I have two objectives.

- I. *First is to give a sense of the extent of accomplishments of these scholars and rishis to the readers. This I will do so by discussing known works.*
- II. *The second, and a continuing quest in these columns is to illustrate one technique of Vedic Math each time.*

BHASKARA’S LILAVATI - A MATHEMATICAL TREATISE

Of the many scholars Bhaskaracharya or Bhaskara II (1114-1193 C.E.) stands out as a teacher and poet. According to the description in his book ‘*Philosophical Crown Jewel*’ [Sidhantashiromani] he lived either in Southern India – probably south of modern day Bombay. Under the able tutelage of his father and teacher Maheshwara a great astronomer, young Bhaskara mastered mathematics, astronomy, Panini (Sanskrit) grammar, and poetry. This treatise written when Bhaskara was 36, consists of four parts: Arithmetic (*Lilavati*), Algebra (*Bijaganita*), Celestial Globe (*Goladhyaya*), and Planetary Mathematics (*Grahaaganita*).

Among these *Lilavati* stands out. The beauty of *Lilavati* is that Bhaskara has been able to distill mathematics into a poetry form with 261 slokas or verses. This great mathematician was an excellent teacher as well, as the two examples below illustrate:

1. In the XVIII’th stanza of *Lilavati* the author says:
*O! you auspicious girl with enchanting eyes of a fawn, Lilavati,
If you have well understood the above methods of multiplication
What is the product of 135 and 12?
Also, tell me what number will you obtain when the product is divided by 12.*
2. In the LIV’th stanza of *Lilavati* the author gives a ‘word problem’:

*Of a group of elephants, half and one third of the half went into a cave,
One sixth and one seventh of one sixth was drinking water from a river.
One eighth and one ninth of one eighth were sporting in a pond full of lotuses
The lover king of the elephants was leading three female elephants; [then], how many elephants were there in the flock?*

Now the reader may be wondering who in the world was ‘*Lilavati*’? According to a 1587 translation by Fyzi (an Arab translator) *Lilavati* was Bhaskara’s daughter. A famed astronomer and astrologer, Bhaskara foresaw that his daughter would not be married and live happily if she is not wedded at an auspicious moment. To find the moment, he constructed a device - a cup with a small hole in its bottom that was placed in a vessel filled with water. The auspicious moment would be when the cup that would sink having slowly filled-up. As fate would have it, on the wedding day, a pearl from *Lilavati*’s dress fell into the cup and blocked the hole and the auspicious moment passed without her getting married. Bhaskara then wrote *Lilavati* to console and detract his grief stricken daughter to whom he taught the mathematical techniques.

As the book demonstrates, Bhaskara though a masterful mathematician, was also a *rasika* as his poetry indicates. He teaches his pupil to be mindful of her surroundings by formulating relevant contextual word problems in arithmetic, algebra and geometry – a clear expert in pedagogy. His book has been used as a standard mathematical text in Indian Gurukulas (traditional schools) for the last eight hundred years.

NIKHILAM SUTRA – PRELUDE TO MULTIPLICATION

To fulfill my second objective, in this column I will illustrate multiplication of two numbers using a *sutra* from Vedic Math called “All from Nine and the last from Ten” (Sanskrit - *Nikhilam Navatashcaramam Dashatah*). I will choose a special case to illustrate this. But, this can be expanded to any multiplication. *The sutra basically means start from the left most digit and begin subtracting ‘9’ from each of the digits; but subtract ‘10’ from the last digit.*

Example 1: Let us choose the number 6. This has only one digit, so it is also the last digit. Thus applying the *Nikhilam sutra* we have 10 subtracted from 6 to get '-4'.

$$\begin{array}{r} 6 \quad \xrightarrow{\text{Nikhilam Sutra}} \quad -4 \\ 87 \quad \xrightarrow{\text{Nikhilam Sutra}} \quad -13 \end{array}$$

Example 2: Given the number 87, it is clear that the first digit is 8 and the last digit is 7. Using the *sutra*: Subtract 9 from 8 to get '-1'; subtract 10 from the last digit 7 to get '-3'. So on the application of the *Nikhilam sutra* we get '-13'.

NIKHILAM APPLICATION: MULTIPLICATION - SPECIAL CASE

In the following examples I will take two numbers and illustrate how to multiply them in a very quick way using *Nikhilam sutra*. Even though this technique works for any pair of numbers, we will look at a special case when the numbers are near a base such as 10, 100, 1000, etc. We start with a simple example.

Example 3: To multiply 8 and 7. Apply *Nikhilam sutra* "All from nine and last from ten" to the number 8 to get '-2' (since there is only one digit so subtract by 10), and for the number 7 to get '-3'. Now write the following:

$$\begin{array}{r} 8 \quad -2 \\ \times 7 \quad -3 \\ \hline \end{array}$$

- Multiply (-2) and (-3) to get '6' and write it down as below.

$$\begin{array}{r} 8 \quad -2 \\ \times 7 \quad -3 \\ \hline 6 \end{array}$$

- Next we '*cross-add*'. That is add 8 and -3 to get '5' **or** add 7 and -2, to get '5' as shown in the picture below. Note that in either of the operations you get the same answer that is '5'.

$$\begin{array}{r} 8 \quad -2 \\ \times 7 \quad -3 \\ \hline 5 \end{array}$$

- We find the solution by combining the numbers we found by the above operations as:

$$\begin{array}{r} 8 \quad -2 \\ \times 7 \quad -3 \\ \hline 5 \quad 6 \end{array}$$

So the answer is '56'.

One interesting observation, the **origin of the multiplication sign** can be traced to the above 'cross-add'ing.

Now you may be wondering that 'I knew the answer all along- big deal'. Well, I used a baby problem to illustrate. I will show you that such multiplication can be done for two and higher digit multiplication.

Example 4: To multiply 92 and 89. Apply *Nikhilam Sutra* - "All from nine and last from ten" on both the numbers.

$$\begin{array}{r} 89 \quad \xrightarrow{\text{Nikhilam Sutra}} \quad -11 \\ 92 \quad \xrightarrow{\text{Nikhilam Sutra}} \quad -08 \end{array}$$

- Write this down side by side.

$$\begin{array}{r} 92 \quad -08 \\ \times 89 \quad -11 \\ \hline \end{array}$$

- Multiply (-08) and (-11) to get '88'.

$$\begin{array}{r} 92 \quad -08 \\ \times 89 \quad -11 \\ \hline 88 \end{array}$$

- Now we cross-add. This is done by either adding 92 and -11 to get '81' **or** adding 89 and -08.

$$\begin{array}{r} 92 \quad -08 \\ \times 89 \quad -11 \\ \hline 81 \end{array}$$

Note that in both operations you get the same answer that is '81' which is written below to get the solution.

$$\begin{array}{r} 92 \quad -08 \\ \times 89 \quad -11 \\ \hline 81 \quad 88 \end{array}$$

So the answer to multiplying '92 × 89' is '8188'.

Again, this technique works very well if the numbers to be multiplied are near a base. Upon slight modification this also works very well for any pair of numbers.

Homework For Fun: Try the "Nikhilam" sutra to multiply: (i) 85×98, (ii) 995×988. (iii) Bonus problem 105×93. Send answers to vedicmath@hotmail.com. All correct answers will be acknowledged.

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